Communicative stability and the typology of logical connectives

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Overview. We argue that the typology of logical connectives can be explained based on a novel notion of communicative stability.

Typology. The typology of logical connectives presents a major cross-linguistic puzzle: Among the 16 binary logical connectives, only 3 are ever lexicalized. Focusing on the 6 connectives which are both commutative and non-trivial (see below), we observe the following pattern:

(1) a. Lexicalization of AND and OR is pervasive and seems to be morphologically simplex.
b. NOR is more rarely lexicalized; when lexicalized, it's morphologically complex.
c. NAND, XOR and IFF are never lexicalized (Horn's puzzle, extended).

	P AND Q	P OR Q	P NOR Q	P NAND Q	P XOR Q	P IFF Q
$p \wedge q$	1	1	0	0	0	1
$p \wedge \neg q$	0	1	0	1	1	0
$q \wedge \neg p$	0	1	0	1	1	0
$\neg p \wedge \neg q$	0	0	1	1	0	1

Previous accounts. Since Horn (1972), explaining the typological pattern has been usually taken to require assuming that 'positive' connectives, e.g., AND, are inherently less complex than 'negative' ones, e.g., NAND (Positivity). As Enguehard and Spector (2021) point out, however, there is no compelling independent evidence for Positivity, and it assumes what needs to be explained (that is, (1a)). Enguehard and Spector propose an account which does not rely on Positivity, but as Züfle and Katzir (2021) argue, their account can only provide a general theory of the typological pattern if Positivity is assumed after all. Furthermore, with the exception of Katzir and Singh (2013), previous accounts of the typology do not explain the complexity of NOR when it's lexicalized, (1b). As for Katzir and Singh, they use an exceptionally strong version of Positivity, and also do not capture the full typological pattern (see Uegaki 2022). Our goal here is to provide an account which does not rely on Positivity and at the same time can fully explain the pattern in (1).

Proposal. The key to the current proposal is the following claim:

(2) **Stability**: Attested languages are languages in which the optimal message for a speaker to choose when they want to convey a particular state is not affected by prior probabilities.

Stability is a desired property for communication: if the speaker's choices don't depend on prior probabilities, it's easier for the hearer to retrieve the speaker's intended state even if the speaker's and hearer's beliefs about the priors differ. In other words it leaves less room for miscommunication. We show that the claim in (2) holds given standard definitions of rational speakers in the Rational Speech Act model (RSA; Frank and Goodman 2012; Bergen et al. 2016), as in its simplified version in (3), assuming that sentences have deletion alternatives (see Sauerland 2004, Fox 2007), as in (4).

(3) a. A naive hearer NH: $P_{NH}(s|m) \propto P(s) \cdot [\![\mathrm{m}]\!](s)$
b. A rational speaker RS: $P_{RS}(m|s) \propto P_{NH}(s|m)$

(4) **The universal availability of deletion alternatives**:
Utterances of the form P *connective* Q have both P and Q as alternatives.

In order to measure the stability of a language we generated all 63 ($= 2^6 - 1$) possible languages containing non-trivial commutative connectives, and randomly assigned prior probabilities 1000 times for each language. We did this twice: once to check stability in positive sentences (e.g., having the

set of alternatives {P AND Q, P, Q} given the inventory {AND}), and another time to check stability in negative sentences (e.g., having the set of alternatives {NEG(P AND Q), NEG(P), NEG(Q)} given the inventory {AND}). A language was considered stable if in both positive and negative cases, the optimal message for a speaker to choose in a state given the resulting speaker's probability distribution P_{RS} was identical across all prior assignments. The only language inventories where speaker's choices turn out to be stable are {AND}, {OR}, and {AND, OR}. Stability then gives us a handle on why AND and OR are the only simple connectives lexicalized.

In order to explain why NOR is the only complex connective lexicalized, we assume the following:

(5) Complex connectives in a language L can only be formed by combining an existing simple connective in L with negation (see Katzir and Singh 2013).

Given that the only simple connectives are AND and OR, the only complex connectives which can be generated given (5) are NAND and NOR. The reason why NOR is the only attested complex connective, we propose, is that NOR is stable in positive contexts while NAND is not. It is reasonable to assume that when complex connectives are at stake what matters is only stability in positive sentences, because NEG(P NOR/NAND Q) would be redundant given the availability of the equivalent and less complex P OR/AND Q (which is guaranteed to exist in the language given (5)). As a result, uttering NEG(P NOR/NAND Q) would be a violation of Gricean Brevity (indeed, NEG(P NOR Q) sentences are rather odd in English). The limited usability of such sentences in negative contexts means it's difficult for them to lead to miscommunication in these contexts.

Rational hearers. Stability can only explain the typology if it leads to good communication, that is if stability in speaker's choices translates into stability in hearer's choices. This does not happen however if we use a standard definition of a rational hearer along the lines of (6): in this case all languages become unstable from the hearer's perspective.

(6) A rational hearer RH: $P_{RH}(s|m) \propto P(s) \cdot P_{RS}(m|s)$

We point out that the problem is due to an issue brought up by Fox and Katzir (2021), which is that RSA models wrongly predict sentences like P OR Q to be interpreted as meaning $p \wedge \neg q$ given some priors. In order to fix the problem and enable stability from the speaker's perspective to translate into stability from the hearer's perspective, we propose letting the rational hearer take into account only the optimal choices of the speaker and ignore sub-optimal ones (cf. Franke 2011).

Trivial and non-commutative connectives. Like previous accounts (see Uegaki 2022), ours does not explain the absence of non-commutative or trivial connectives. Trivial connectives can be ruled out given a ban on the use of trivial expressions (see Gajewski 2002). The absence of non-commutative ones seems to require assuming that connectives must be defined based on functions from *sets* of truth values, rather than tuples (see Gazdar and Pullum 1976).

Extension to quantifiers. Parallel patterns to the one in (1) have been observed with quantificational determiners and modals. Extending our analysis to these cases requires having parallel sets of alternatives. Focusing on nominal quantifiers, a sentence like *Det kid smiled* should have the alternatives {*Mary smiled*, *John smiled*} if the kids are Mary and John. We propose that such alternatives are generated by deleting the quantifier phrase as well as the (highest) lambda abstraction over the variable in its scope. This yields an expression with an assignment dependent variable; the alternative propositions result from all possible assignments for this variable, as in (7). Note that for existential and universal quantifiers, this results in subdomain alternatives (Chierchia 2013, a.o.). Accounting for the typological pattern with modals can be done in a similar fashion.

(7) $Alt(\text{QP } [\lambda 1\ [\ldots t_1 \ldots]]) \supseteq \{[\![[\ldots t_1 \ldots]]\!]^{g(1)\to x} : x \in D_e\}$

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