Proportional Comparatives and Relative Scales

Stephanie Solt (ZAS Berlin)

Issue: While most work on the semantics of measurement has focused on absolute measures such as in (1), there has recently been some attention paid to relative or proportional measures, as in (2) (Ahn & Sauerland 2015, Kotek et al. 2015):

(1) a. <u>2 kilos</u> of (the) rice b. <u>10 liters</u> of (the) water c. <u>six</u> cats

(2) a. <u>20 percent</u> of the students b. <u>three quarters</u> of Americans

The usual assumption is that the underlying measurement scale in the relative cases is the number line (i.e. the natural, rational or real numbers), with the calculation of the proportional relation lexicalized by the relative expression itself. For example, Ahn & Sauerland offer the following as the semantics of *percent*:

(3) $[\![percent]\!] = \lambda x_e\, \lambda n_d\, \lambda y_{et}\,.\ \dfrac{\mu(x \sqcap \bigoplus y)}{\mu(x)} = \dfrac{n}{100}$

However, there is evidence that the semantics makes use of scales that directly track proportions. This is seen most clearly in examples such as (4), first noted by Partee (1989). The number of residents of Ithaca (population 30,000) who know their neighbors is undoubtedly smaller than the number of residents of NY (population 8 million) who do so, and likewise the number of Brazilians who have degrees is larger than the number of Norwegians that do. Nonetheless, both of these sentences have a reading on which they are true; on this reading, it is proportions rather than absolute numbers that are compared.

(4) a. More residents of Ithaca than New York City know their neighbors.
b. Fewer Brazilians than Norwegians have university degrees.

Adopting a standard semantics for the comparative as in (5), and making the usual assumption that μ is a cardinality function, we derive only the false reading. But the true reading can be obtained if we allow μ to be a proportion measure function, i.e. a function that maps pluralities to the proportion they represent of the relevant totality.

(5) $max(\lambda d.\exists x[{*}Ithaca\text{-}resident(x) \wedge \mu(x)=d \wedge knows\text{-}neighbors(x)]) \succ$
$max(\lambda d.\exists x[{*}NYC\text{-}resident(x) \wedge \mu(x)=d \wedge knows\text{-}neighbors(x)])$

Further evidence for the existence of degrees of proportions comes from distributional parallels between proportional expressions such as *20 percent* and numerals. The latter occur in environments where they are most naturally analyzed as denoting degrees, a prime example being mathematical statements (Rothstein 2012), as in (6a); percentage expressions occur in these same environments, per (6b). Similarly, in differential comparatives such as (7), the numeral or percentage must have a degree-based interpretation, as there is no entity or plurality to serve as the argument of a measure function (cf. Solt 2015). Such examples are not captured by the relational entry (3).

(6) a. Two plus two is four.
b. Twenty percent plus twenty percent is forty percent.

(7) There were 200/20% fewer houses sold this month than last month.

Proposal: In formalizing the above observations, I begin with the partitive cases in (2), where proportional measurement is explicit. Following Ionin et al. (2005) I assume that partitive *of* introduces a 'part of' relation, and drawing on Schwarzschild (2006) I take it to also encode measurement. Thus the lexical entry for *of* is that in (8). The first argument of *of* is an individual (type *e*). In partitives such as *ten/20% of the students*, this is provided by the definite description, per (9); in pseudopartitives such as *20% of Americans*, I follow Ahn & Sauerland in assuming a maximal entity is derived from the predicative interpretation of the noun phrase via a covert supremum operator.

(8) $[\![of]\!] = \lambda x_e\, \lambda d_d\, \lambda y_{et}.\, y \sqsubseteq x \wedge \mu_x(y)=d$
(9) $[\![of\ the\ students]\!] = \lambda d_d\, \lambda y_{et}.\, y \sqsubseteq the\text{-}students \wedge \mu_{the\text{-}students}(y)=d$

Here μ_x is an underspecified measure function, which maps component parts of the individual x to degrees. Depending on context, these might be degrees of weight (*300 grams of the fish*), volume (*3 liters of the wine*) or cardinality (*10 of the students*). I propose that degrees corresponding to proportions of x (in terms of weight/volume/ cardinality/etc.) are simply another possibility. Thus in an example of the form *20% of the students*, the measure function is $\mu_{the\text{-}students}(y) = |y| \,/\, |the\text{-}students|$, i.e. a function that maps subparts of the totality of relevant students to their proportion of that totality.

As further support for the claim that proportional partitives involve mapping to proportions, these – but not absolute measure partitives – allow the lower noun phrase to be modified by *all*:

(10) 20 percent/three quarters/*twenty/*20 kilos of all (the) potatoes harvested

Following observations by Partee (1995) and others, I take *all* in such cases not to be a quantifier, but instead to merely add a meaning of exhaustiveness to the ordinary meaning of the definite description or bare plural. This has a semantic effect in the proportional case, in that it removes potential ambiguity as to the totality on which proportions are calculated, but would be superfluous in the absolute case.

We turn now to proportional comparatives such as (4). I assume these to have the same logical form as ordinary absolute quantity comparatives (11), and that as in the absolute case, degrees of 'quantity' are introduced via a null functional head *Meas* (cf. Solt 2015). Differing from the absolute case, I take *Meas* in the proportional case to be the covert counterpart of *of* in (8); this assumes that the denotation of the noun phrase shifts to a type *e* denotation via a supremum operator (parallel to *20% of Americans*). Saturating the first argument of *Meas* yields (12a); as in the partitive case, μ can be interpreted as a proportion function, per (12b). Assuming a parallel interpretation for *Meas* in the *than*-phrase, we correctly derive a comparison of proportions.

(11) [-er than [2 [t2 Meas NYC residents ~~know neighbors~~]]][1 [t1 Meas Ithaca residents know neighbors]]

(12) a. $[\![Meas\ Ithaca\ residents]\!] = \lambda d \lambda y.\, y \sqsubseteq sup(Ithaca\text{-}residents) \wedge \mu_{Ithaca\text{-}residents}(y)=d$
b. $\mu_{Ithaca\text{-}residents}(y) = |y| \,/ |sup(Ithaca\text{-}residents)|$

Conclusions and extensions: The broader conclusion from this analysis is that dimensions such as cardinality can be tracked by multiple scales; one possible scale type is a scale that represents proportions of a totality rather than absolute values. I will discuss two additional cases of proportional or relative scales. The first has to do with the quantifiers *many/few*: while these are known to allow both cardinal and proportional readings (Partee 1989), in certain contexts, including partitives and with individual level predicates, only the proportional reading is possible. I will argue that these necessarily involve mapping to a proportional scale. A second case involves adjectival examples of the form *(*5 feet) tall for a boy*, which Bale (2011) analyzes in terms of a scale derived relative to a comparison class.

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