A surface-scope analysis of authoritative readings of modified numerals

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Overview. This paper concerns a well-known puzzle regarding the interpretation of superlative modified numerals in the scope of existential modals. We present new data that broaden the scope of the puzzle, and we offer a new, surface-scope solution based on a conservative amendment of the recursive exhaustification approach of Fox 2007.

Basic puzzle. A well-known puzzle regarding the interpretation of *at most n* is that when it occurs in the scope of an existential deontic modal, as in (1a), the resulting sentence can have an *authoritative reading*, characterized by two kinds of inferences: (i) an upper-bound (UB) inference, viz. that you're not allowed to draw 4 or more cards, and (ii) a free-choice (FC) inference, viz. that you're allowed to draw any number of cards in the range [0, 3] (Geurts and Nouwen 2007; Büring 2008; Nouwen 2010; Penka 2014; Kennedy 2015). On standard assumptions about the meanings of *allow* and *at most 3*, however, a surface-scope analysis of (1a) predicts only a weak literal meaning: there is some permissible world in which you draw 3 cards or fewer ($\Diamond[\leq 3]$); neither UB nor FC logically follow from this. ((1a) also has an *ignorance reading*, which we ignore here; this is the reading that would result from inverse scope; Penka 2014; Kennedy 2015.)

(1) a. You're allowed to draw at most 3 cards.
  b. #You're allowed to draw at least 5 cards.

Curiously, *at least n* does *not* give rise to an analogous authoritative reading: (1b) cannot be used to convey that you're allowed to draw 5 or more cards, with FC in the range [5, ...], and a lower bound (LB) that prohibits drawing 4 cards or fewer; thus, in contexts that do not support an ignorance reading, (1b) is infelicitous (hence the '#'). The puzzle, then, has two parts: (i) why does (1a) have an authoritative reading, and (ii) why does (1b) not?

A previous analysis. Penka (2014), quite reasonably, takes *at most* to be the 'oddity' in this puzzle and solves it by decomposing *at most* into a negative component, *ANT*, plus *at least*, so that (1a) is analyzed with split scope as *ANT 3 [... allow [... at least ...]]*, which means 'you're not allowed to draw more than 3 cards' ($\neg \Diamond [\geq 4]$), thus entailing an UB. The FC inferences follow from neo-Gricean reasoning, given certain assumptions about the scales responsible for generating alternatives. That (1b) has no authoritative reading, in particular no LB, follows because *at least* is assumed to have no analogous negative component: *allow [... at least 3 ...]* receives only a weak interpretation.

New observations. The novel data in (2) show that a variety of expressions, beyond *at most n*, can have authoritative readings. For instance, (2a) shows that two 'antonyms' can each have authoritative readings with opposite bound inferences, viz. that you may not arrive {earlier/later} than 8:00 PM (in addition to their FC inferences, viz. that you may arrive at 8:00 PM and you may arrive {later/earlier}). (2b) exhibits double-boundedness, as well as FC effects within the allowable range, viz. that you can borrow any amount between $1,000 and $5,000, but you may not borrow less than $1,000 or more than $5,000. Finally, (2c) shows that, with the definite article, the lower-bound superlative modifier can indeed convey a LB authoritative reading.

(2) a. You may arrive {at the earliest / at the latest} at 8:00 PM.
  b. You're allowed to borrow between $1,000 and $5,000 from this bank.
  c. You can have at the least 3 children (and still qualify for the tax exemption).

Penka's approach cannot capture these authoritative readings without *ad hoc* assumptions, e.g. that *at the earliest* decomposes into 'not earlier than', while *at the latest* decomposes into 'not later than'; that *between m and n* decomposes into 'not fewer than m and not more than n'; and that *at the least* (but not *at least*) decomposes into 'not fewer than'. Note also that an inverse-scope analysis would predict ignorance inferences across the board. The conclusion we draw is that it is not *at most* which is ill-behaved, nor *at least* (per se), but rather the modified numeral *at least n*. Given that disjunction in the scope of an existential modal licenses similar FC and exclusivity inferences, and has received a surface-scope analysis based on (recursive) exhaustification (Fox 2007), it seems worthwhile to try to extend such an analysis to these cases as well.

Proposal. We propose a surface-scope analysis of the authoritative reading of a sentence S of the form … *allow* … *at most 3* …, in which the literal meaning $\Diamond[\leq 3]$ is strengthened by recursive exhaustification into $[\![$*exh [exh S]*$]\!] = \Diamond[\leq 3] \wedge \neg\Diamond[\geq 4] \wedge \Diamond[=3] \wedge \Diamond[=2] \wedge \cdots$. For the first (inner) round of exhaustification, we assume that the set of alternatives to S ($\text{alt}(S)$) includes those obtained by replacing *at most* with *exactly* and *3* with any numeral, i.e. $\{\Diamond[=n] \mid n \in \mathbb{N}\} \subseteq \text{alt}(S)$. All innocently excludable (IE) alternatives in $\text{alt}(S)$ are then excluded in the standard way (Fox 2007), which derives an UB, because $\Diamond[=4]$, $\Diamond[=5]$, etc. are all IE. For the second (recursive/outer) round of exhaustification, if we assumed, as Fox (2007) does, that $\text{alt}(exh\ S)$ is the set of all strengthened alternatives to S, i.e. $\{\text{exh}(\text{alt}(S))(p) \mid p \in \text{alt}(S)\}$, then it turns out that we would not derive strong enough FC inferences: for instance, *exh [exh S]* would be true in any scenario in which exactly 1 and exactly 3 are allowed, but exactly 2 (and more than 3) is forbidden. Instead, we propose that the set of alternatives for the second exhaustification includes not simply all strengthened propositions taken from alt(S), but rather all strengthened propositions taken from the *disjunctive closure* of $\text{alt}(S)$, i.e. $\text{alt}(exh\ S) = \{\text{exh}(\text{alt}(S))(p) \mid p \in \text{alt}(S)^{\vee}\}$. The effect of this amendment is to introduce *weaker* propositions into the alternative set, so that their exclusion results in stronger inferences. For example, $p = \Diamond[=0] \vee \Diamond[=1] \vee \Diamond[=3]$ is in $\text{alt}(S)^{\vee}$. $\text{exh}(\text{alt}(S))(p)$ entails $\neg\Diamond[=2]$; hence, negating $\text{exh}(\text{alt}(S))(p)$, together with the strengthened assertion $\text{exh}(\Diamond[\leq 3]) = \Diamond[\leq 3] \wedge \neg\Diamond[\geq 4]$, entails $\Diamond[=2]$. Thus, the overall meaning derived is $\Diamond[\leq 3]$ (basic meaning), plus $\neg\Diamond[\geq 4]$ (first *exh*: UB), plus $\Diamond[=3] \wedge \Diamond[=2] \wedge \Diamond[=1] \wedge \Diamond[=0]$ (second *exh*: FC).

Importantly, our amendment does not disrupt the analysis of FC disjunction, because in this case the alternative set is already closed under disjunction. In addition, our proposal extends naturally to all the cases in (2). However, it also incorrectly predicts a LB authoritative reading of (1b). Thus, if our proposal is on the right track, then, together with our new empirical observations, it brings to light a new angle on an old puzzle: What prevents *at least n* from participating in authoritative readings under an existential modal?

Extensions. Since *exh* can be embedded, we predict authoritative readings to arise in the scope of other operators. By contrast, Penka's neo-Gricean account predicts only bound inferences in such environments; FC ought to disappear. That a sentence like *Everyone is allowed to submit at most 3 letters* has a natural reading in which everyone has FC in the range [0, 3] appears to support our surface-scope, exhaustivity-based account.

Finally, note that bare numerals also give rise to UB readings (*You're allowed to take 3 classes*). Assuming that numerals are simplex expressions with 'at least' meanings, and hence do not evoke 'exactly' alternatives, our account predicts this via surface scope: for $\Diamond[\geq 3]$, we exclude $\Diamond[\geq 4]$. This analysis is thus an alternative to Kennedy's (2015) and, importantly, evades the criticisms he levies against neo-Gricean approaches.

References

Büring, Daniel (2008). The Least *at least* Can Do. In: *Proceedings of the 26th West Coast Conference on Formal Linguistics*. Ed. by Charles B. Chang and Hannah J. Haynie. Somerville, MA: Cascadilla Proceedings Project, pp. 114–120. URL: http://www.lingref.com/cpp/wccfl/26/paper1662.pdf.

Fox, Danny (2007). Free Choice and the Theory of Scalar Implicatures. In: *Presupposition and Implicature in Compositional Semantics*. Ed. by Uli Sauerland and Penka Stateva. Palgrave Studies in Pragmatics, Language and Cognition Series. Palgrave Macmillan. Chap. 4, pp. 71–120. DOI: 10.1057/9780230210752_4.

Geurts, Bart and Rick Nouwen (2007). At least et al.: The semantics of scalar modifiers. In: *Language* 83, pp. 533–559. URL: http://www.jstor.org/stable/40070901.

Kennedy, Chris (2015). A "de-Fregean" semantics (and neo-Gricean pragmatics) for modified and unmodified numerals. In: *Semantics and Pragmatics* 8.10, pp. 1–44. DOI: 10.3765/sp.8.10.

Nouwen, Rick (2010). Two kinds of modified numerals. In: *Semantics and Pragmatics* 3.3, pp. 1–41. DOI: 10.3765/sp.3.3.

Penka, Doris (2014). *At most* at last. In: *Proceedings of Sinn und Bedeutung 19*. Ed. by Eva Csipak and Hedde Zeijlstra, pp. 463–480. URL: http://nbn-resolving.de/urn:nbn:de:bsz:352-0-304168.